A NEW AUDIO-FREQUENCY BRIDGE

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ABSTRACT. A new bridge for the measurement of the frequency of an alternating current in the audio-frequency range has been devised, in which the null point is obtained virtually by the adjustment of the resistance in one of the arms; the adjustment characteristic is given by $f = a\sqrt{x}$ which is a parabola The vector diagrams of the balanced bridge have been constructed and discussed.

INTRODUCTORY

The basic principles underlying the many available methods for measuring the frequency of an alternating current are very varied but they have been classified under six broad headings by A. Campbell and E. Childs[1]; the bridge method is one of these six groups. This group comprises those instruments in which an alternating voltage is applied between two points of an electrical network and by proper adjustment of the values of capacity, resistance and inductance (self or mutual) in the network, the potential difference between two other points in the circuit is reduced to zero, as indicated by a detecting instrument, such as a telephone or a vibration galvanometer, connected across these points. If one of the adjustments depends on the frequency of the alternating current then the bridge may be used as a measurer of frequency. It is desirable that no other adjustment should be necessary but this has not been really attained in most of the systems. Now these frequency bridges may be further subdivided into the following sections according to the nature of the elements used in the arms of the bridge :—

(*a*) Network containing R and L [2, 3]
(*b*) ,, ,, R and C [2, 3, 4, 5]
(*c*) ,, ,, R, L and C [2, 3, 6, 7, 8]
(*d*) ,, ,, R, L and M [9, 10, 11, 12, 13, 14, 15]
(*e*) ,, ,, R, L, M and C [16, 17, 18, 19, 20, 21, 22]

where R, L, M and C stand respectively for resistance, self-inductance, mutual-inductance and capacity. A useful summary of the methods is given by J. Kronert [23] and also by B. Hague.[24] H. Mukherjee [25] devised a Wheatstone network containing capacity and resistance, and put the moving coil of a dynamometer type A. C. galvanometer in one of diagonal arms and the fixed coils in the other in series with the current source ; the elements in the arms were adjusted till the galvanometer showed no deflection ; the value of the frequency was obtained in terms of resistance and capacity. On the other hand, the frequency bridges may be classified according to their adjustment characteristics. If x is the adjustment of the bridge entering into the frequency balance condition, we can write for the adjustment characteristic of the bridge, f=function (x). B. Hague[21] has graphically described eleven adjustment characteristics covering all the frequency bridges used in practice. The new bridge, a short account of which was published in *Current Science*,[26] has an adjustment characteristic given by $f = a\sqrt{x}$ which is a parabola.

THE THEORY AND DESCRIPTION OF THE BRIDGE

Fig. 1 is the circuit diagram of the new bridge. The branch AC of the bridge contains the primary coil of the mutual inductor M, the resistance and self-inductance L of the primary coil being known ; in series with it an adjustable non-inductive resistance box is connected ; P denotes the total resistance including that of the coil in the branch AC. The arms BC and AD contain adjustable non-inductive resistance boxes only ; K is a good mica condenser in the branch DB.

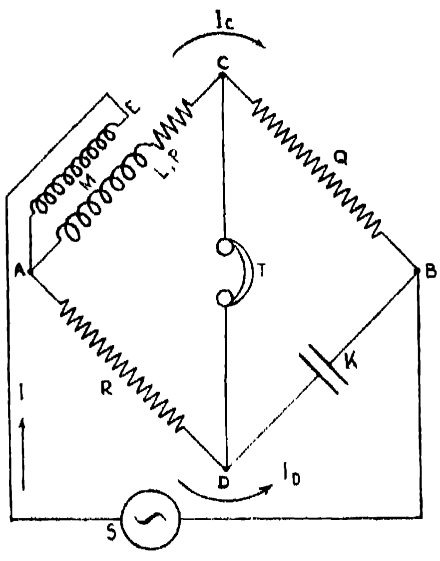

FIG. 1

Circuit diagram

One end of the secondary coil of the mutual inductor M is connected to that of the primary at A and the other end is connected to one terminal of the source S of the alternating current whose frequency has to be measured ; the other terminal of the source is connected to the point B, the junction of Q and K. T is a telephone used as a detector of the balanced state.

Let I_c be the vector representing the harmonically varying current of frequency f per sec., flowing along the branch ACB when the bridge is balanced, I_b that along ADB, and I that along BSEA ; no current flows along the branch CD at balanced state. Applying Kirchhoff's rules to the meshes of the impedance network carrying harmonic alternating current we get the following equations :—

At A $$I = I_c + I_b \; ; \qquad \ldots \quad (1)$$

for the mesh ACD,

$$(P + j\omega L) I_c - R I_b + j\omega M I = 0, \qquad \ldots \quad (2)$$

where $\omega = 2\pi \times$ frequency, and $j = \sqrt{-1}$;

for the mesh CBD,

$$Q I_c - \left(\frac{-j}{k\omega} \right) I_b = 0. \qquad \ldots \quad (3)$$

Substituting the equation (1) in (2) and rearranging the equations (2) and (3) we get

$$(P + j\omega L + j\omega M) I_c = (R - j\omega M) I_b \; ; \qquad \ldots \quad (4)$$

and $$Q I_c = \frac{-j}{k\omega} . I_b. \qquad \ldots \quad (5)$$

Dividing (4) by (5) we have

$$\frac{P + j\omega(L + M)}{Q} = \frac{R - j\omega M}{-\frac{j}{k\omega}}$$

or $$-\frac{j}{k\omega} \{P + j\omega(L + M)\} = QR - j\omega MQ \; ,$$

or $$-\frac{jP}{k\omega} + \frac{L + M}{k} = QR - j\omega MQ.$$

Equating the real quantities and likewise the unreal quantities we get

$$\frac{L + M}{k} = QR \qquad \ldots \quad (i)$$

and $$\frac{P}{k\omega} = \omega MQ$$

or $$\omega^2 = \frac{P}{kMQ} . \qquad \ldots \quad (ii)$$

$$\therefore \quad f = \sqrt{P} . \; (2\pi \sqrt{kMQ})^{-1}.$$

Hence the conditions (*i*) and (*ii*) are to be satisfied for no current in the telephone. The condition (*i*) may be secured by varying R, and the condition (*ii*) by varying P.

In practice, at first an audio-frequency alternating current is supplied to the bridge, and L, M, K and Q are maintained fixed; balance is obtained by successive adjustment of P and R. The process is easy and rapidly convergent since the two conditions of balance are mutually independent. When the proper values of P and R for balance at any frequency have been determined, the first condition, which is independent of frequency, remains satisfied for all frequencies, and the frequency bridge is thus brought to working order; we may now measure frequencies of alternating current sources by varying P alone so that virtually a single adjustment is all that is necessary in order to determine the frequency. The formula representing the frequency f may be written $f = a\sqrt{P}$, where $a = (2\pi\sqrt{KMQ})^{-1}$ = a constant; thus the adjustment characteristic is a parabola.

The vector diagram of the balanced bridge is constructed and discussed below. In Figs. 2*a* and 2*b* let the vector AB denote the voltage e across

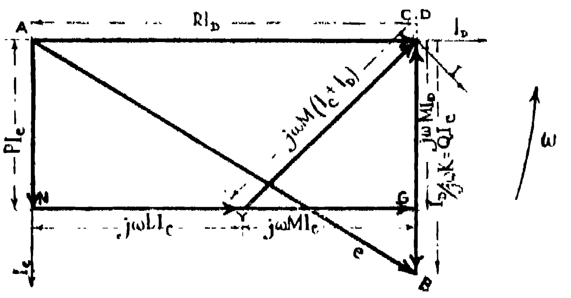

FIG. 2*a*

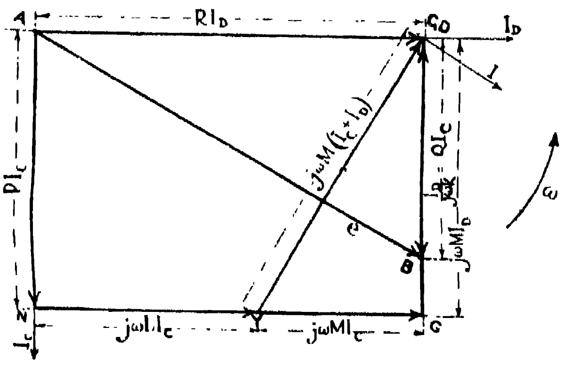

FIG. 2 *b*

the points A and B of the circuit diagram. The current I_C flowing along the inductive branch ACB lags behind e, while the current I_D flowing along the capacitative branch ADB leads e; RI_D is the potential drop across the resistance R and is represented by the vector AC or AD, since C and D are equipotential points when the bridge is balanced; the potential differences across the condenser K is $I_D/j\omega k$ and is represented by DB at right angles to AC, thus AC + CB = AB, that is, $RI_D + \frac{I_D}{j\omega k} = e$. Now the potential drop across the resistance Q is QI and is represented by CB or DB; therefore $QI_c = \frac{I_D}{j\omega k}$. The vector $AN = PI_c$ represents the potential drop in the resistance P of the arm AC of the circuit diagram; this vector is in phase with the current I_c; at right angles to this is the vector NY which represents the potential drop in the inductance L and is equal to $j\omega LI_c$; in phase with NY is the vector YG representing $j\omega MI_c$; the vector GD represents $j\omega MI_D$; thus $YD = YG + GD = j\omega M(I_c + I_D)$; and we notice that AD = AN + NY + YD; in other words, $RI_D = PI_c + j\omega LI + j\omega M(I_c + I_D)$; and finally AB = AN + NY + YD + DB or $e = PI_c + j\omega LI_c + j\omega M(I_c + I_D) + QI_c$; at right angles to YC is the vector of I, the total current drawn from the source. From the geometry of the vector diagrams it easily follows that

$$j\omega(L+M)I_c = RI_D \qquad \ldots \quad (\alpha)$$

$$PI_c = -j\omega MI_D \qquad \ldots \quad (\beta)$$

$$QI_c = \frac{I_D}{j\omega k}. \qquad \ldots \quad (\gamma)$$

Dividing (α) by (γ) we get

$$\frac{j\omega(L+M)}{Q} = R \,.\, j\omega k$$

$$\text{or} \quad \frac{L+M}{K} = QR,$$

which is the same as the equation (i) for the balaced condition.

Dividing (β) by (γ) we get

$$\frac{P}{Q} = \omega^2 MK$$

$$\text{or} \quad \omega^2 = \frac{P}{KMQ},$$

which is the same as the equation (ii) for the balanced condition.

Thus the two conditions for balance are deduced from the vector diagram. It may be noted that the currents I_c and I_D are at quadrature. It will be further observed from a study of the Figs. $2a$ and $2b$ that when P becomes equal to Q,

$$AN = CB = CG;$$

$$\therefore \frac{I_D}{j\omega k} = -j\omega MI_D \qquad \text{or} \quad \omega^2 = \frac{1}{MK};$$

that is, the second condition for balance is reduced to a simpler form $\omega^2 = \frac{1}{MK}$ when P = Q.

In deducing the balanced conditions it has been assumed that the resistances P, Q and R are perfect, *i.e.*, they contain no residual inductance and capacitance;

this is very nearly true in the case of resistances generally used in the alternating current experiments ; the small residuals may be ignored. Since there is no other coil offering self-inductance, excepting the primary of the mutual-inductor, in any arm of the bridge there is no necessity of arranging the elements in the arms with special precautions so as to avoid the stray mutual-inductance effect. It may be pointed out that the effects of the impurities in the mutual-inductor M and in the mica condenser K have been ignored all together in deducing the conditions for balance ; ordinariiy these are very small and may be neglected ; it is desired to take into account these effects and their influence on the balance conditions later in a subsequent paper.

EXPERIMENTAL EXAMPLE AND REMARKS

The new frequency bridge was used to calibrate a triode oscillator meant to serve as a source for alternating current bridge measurements. The frequency could be varied by changing the capacity of the anode condenser. In the bridge set up, M was 80·5 milli-Henries from a Campbell mutual-inductometer, the fixed winding of which had an inductance $L=98{\cdot}59$ milli-Henries , with $Q=370$ ohms, $R=487$ ohms, which were determined once for all in the beginning as explained above, and $K=1$ micro farad, the balance was obtained by adjusting P with the following results ; the detector used was a telephone ; for the sake of comparison the corresponding values were also determined by the Campbell's frequency bridge and have been recorded in the last column of the table below.

Capacity of the Anode condenser		P		f in cycles per second	f in cycles/sec. as determined by Campbell's Bridge
2.1	micro-farad	190	ohms	402	398
1.6	,,	245	,,	456	452
1.1	,,	364	,,	556	552
0.9	,,	441	,,	614	609
0.7	,,	570	,,	696	692
0.5	,,	795	,,	822	827
0.3	,,	1330	,,	1064	1059
0.1	,,	2000	,,	1301	1293
0.06	,,	2470	,,	1449	1441
0.03	,,	3060	,,	1613	1616
0.01	,,	3640	,,	1759	1758
0.005	,,	3810	,,	1800	1802

If the alternating current is impure the harmonics may cause some trouble in securing a good balance, particularly if the experimenter has a bad ear for distinguishing one pitch from the other. This difficulty is common to all null methods for measuring frequency of an alternating current, but with a little practice most observers may learn to ignore harmonics. The difficulty does not arise at the lower frequencies if a vibration galvanometer is employed instead of a telephone. With telephones as detectors a good balance is often attained by earthing the point D (see Fig. 1) of the circuit thus bringing C and D at balance at the potential of the observer.

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